On the decomposition of the Hochschild cohomology group of a monomial algebra satisfying a separability condition

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This is a joint work with T. Furuya and K. Sanada.

Let k be an algebraically closed field and Q a finite connected quiver. Then kQ denotes the path algebra of Q over k. If an admissible ideal I of kQ is generated by a finite number of paths in Q, we call $\Lambda = kQ/I$ a monomial algebra.

In [B], for a monomial algebra Λ, Bardzell determined a minimal projective resolution of Λ as a right Λ^{e}-module using the idea of an associated sequence of paths introduced in [GHZ]. Therefore, by using this resolution, it is possible to calculate the Hochschild cohomology groups $\mathrm{HH}^n(\Lambda) := \mathrm{Ext}^n_{\Lambda^{\mathrm{e}}}(\Lambda, \Lambda)$ $(n \geq 0)$, where $\Lambda^{\mathrm{e}} := \Lambda^{\mathrm{op}} \otimes_k \Lambda$ is the enveloping algebra of Λ.

This talk is based on [IFS]. In this talk, we consider a finite connected quiver Q having two subquivers $Q^{(1)}$ and $Q^{(2)}$ with $Q = Q^{(1)} \cup Q^{(2)} = (Q_0^{(1)} \cup Q_0^{(2)}, Q_1^{(1)} \cup Q_1^{(2)})$. Let $\Lambda = kQ/I$, $\Lambda_{(1)} = kQ^{(1)}/I^{(1)}$ and $\Lambda_{(2)} = kQ^{(2)}/I^{(2)}$, where I is a monomial ideal of kQ and $I^{(i)}$ is a monomial ideal of $kQ^{(i)}$ for $i = 1, 2$. We assume that I and $I^{(i)}$ $(i = 1, 2)$ are admissible ideals. For any $n \geq 2$, $AP(n)$ denotes the set of paths obtained by linking the associated sequence of paths as defined in [B] and [GHZ], where we set $AP(0) = Q_0$ and $AP(1) = Q_1$. Similarly, $AP^{(i)}(n)$ denotes the set of paths obtained by linking the associated sequence of paths for $i = 1, 2$. For the monomial algebra Λ, under a separability condition $AP^{(1)}(1) \cap AP^{(2)}(1) = \varnothing$ introduced in [IFS], we investigate a relationship between the minimal projective bimodule resolution of Λ given by Bardzell ([B]) and that of $\Lambda_{(i)}$ $(i = 1, 2)$. Moreover, we show that, for $n \geq 2$, the Hochschild cohomology group $\mathrm{HH}^n(\Lambda)$ of Λ is isomorphic to the direct sum of the Hochschild cohomology groups $\mathrm{HH}^n(\Lambda_{(1)})$ and $\mathrm{HH}^n(\Lambda_{(2)})$.

References

[B] M. J. Bardzell, The alternating syzygy behavior of monomial algebras, J. Algebra **188** (1997), no. 1, 69–89.

[EH] K. Erdmann and T. Holm, Twisted bimodules and Hochschild cohomology for self-injective algebras of class A_n, Forum Math. **11** (1999), no. 2, 177–201.

[FS] T. Furuya and N. Snashall, Support varieties for modules over stacked monomial algebras, Comm. Algebra **39** (2011), no. 8, 2926–2942.

[GHZ] E. L. Green, D. Happel and D. Zacharia, Projective resolutions over Artin algebras with zero relations, Illinois J. Math. **29** (1985), 180–190.

[GSS] E. L. Green and N. Snashall, The Hochschild cohomology ring modulo nilpotence of a stacked monomial algebra, Colloq. Math. **105** (2006), no. 2, 233–258.

[GSS] E. L. Green, N. Snashall and Ø. Solberg, The Hochschild cohomology ring modulo nilpotence of a monomial algebra, J. Algebra Appl. **5** (2006), no. 2, 153–192.

[IFS] A. Itaba, T. Furuya and K. Sanada, On the decomposition of the Hochschild cohomology group of a monomial algebra satisfying a separability condition, to appear in Comm. Algebra.

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