SPECTRAL STRUCTURES OF GROTHENDIECK CATEGORIES

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In this talk, we investigate Grothendieck categories by introducing associated topological spaces which we call the *atom spectra* of Grothendieck categories. The class of Grothendieck categories includes many important categories: for example, the category of linear representations of a group, the category of modules over a ring, the category of sheaves of modules over a topological space, and the category of quasi-coherent sheaves on a scheme. Moreover, the Gabriel-Popescu theorem ([PG64, Proposition]) claims that every Grothendieck category is equivalent to some quotient category of the category of modules over some ring. This means that each Grothendieck category reflects some categorical aspects of rings. Therefore the study of Grothendieck categories are essential in representation theory of rings.

The atom spectrum $\operatorname{ASpec}\mathcal{A}$ of a Grothendieck category $\mathcal{A}$ is a generalization of the prime spectrum $\operatorname{Spec}R$ of a commutative ring R. Indeed, we have a canonical bijection between $\operatorname{ASpec}(\operatorname{Mod}R)$ and $\operatorname{Spec}R$, where $\operatorname{Mod}R$ denotes the category of R-modules. In general, the atom spectrum $\operatorname{ASpec}\mathcal{A}$ has a topology. In the case of a commutative ring R, the open subsets of $\operatorname{ASpec}\mathcal{A}$ correspond to the specialization-closed subsets of $\operatorname{Spec}R$. By using this topology, we give the following classification of the localizing subcategories in terms of the atom spectrum, which is shown by Gabriel [Gab62] in the case of commutative noetherian rings.

Theorem 1 ([Kan12a, Theorem 5.5])**.** *Let $\mathcal{A}$ be a locally noetherian Grothendieck category. Then there exists a bijection between the localizing subcategories of $\mathcal{A}$ and the open subsets of* $\operatorname{ASpec}\mathcal{A}$.

For a Grothendieck category $\mathcal{A}$, the atom spectrum $\operatorname{ASpec}\mathcal{A}$ is a Kolmogorov space. Hence it has the partial order called *specialization order*. The specialization order on the atom spectrum is a generalization of the inclusion relation of prime ideals of a commutative ring. By using examples, we see some phenomena of atom spectra of Grothendieck categories which do not occur in the case of commutative rings.

References

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