THE SPACE OF KNOTS IN A MANIFOLD AND AN A_∞-RIGHT MODULE OF CONFIGURATION SPACES

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In this talk, we introduce a spectral sequence converging to the singular homology of $Emb(S^1, M)$, the space of smooth embeddings from the circle to a closed simply connected smooth manifold M of dimension ≥ 4. Such a spectral sequence was constructed by Vassiliev [1]. The relationship between Vassiliev's spectral sequence and ours is still unclear but if they are isomorphic at some page, our spectral sequence would give a conprehensive description for Vassiliev's one and enable us to study it in terms of algebraic topology.

Let STM^n denote n-times direct product of the total space of the sphere tangent bundle of M, and $STM^n|_X$ the total space of the restriction of the base to a subspace $X \subset M^n$. Let $F_nM \subset M^n$ denote the configuration space of n-ordered points in M, and $D_nM \subset M^n$ the fat diagonal of M^n. Let $\mathcal{M}_M$ denote the A_∞-operadic right module of configuration spaces of points with a tangent vector in M, and $\mathbb{R}Map(-,-)$ the derived mapping space between two right modules.

The right module $\mathcal{M}_M$ and the space $Emb(S^1, M)$ is related by a weak homotopy equivalence $Emb(S^1, M) \simeq \mathbb{R}Map(\mathcal{M}_{S^1}, \mathcal{M}_M)$ due to Boavida-Weiss [2] and Turchin [3]. For $\mathcal{M}_M$, we prove the following claim, an enriched version of the Poincaré-Lefschetz duality $H_*(STM^n|_{F_nM}) \simeq H^*(STM^n, STM^n|_{D_nM})$:

Theorem 1. *Let M be a closed orientable smooth manifold. In the category of symmetric spectra, the dual comodule of $\mathcal{M}_M$ is weakly equivalent to a comodule C_M consisting of certain Thom spectra associated to a vector bundle over $STM^n/(STM^n|_{D_nM})$.*

The comodule C_M in Theorem 1 has a Čech-type resolution, and it induces the desired spectral sequence:

Theorem 2. *Let M be a closed simply connected smooth manifold of dimension ≥ 4. There exists a spectral sequence converging to the singular homology $H_*(Emb(S^1, M))$ whose E^2-page is described by the homology groups of a fiber product $STM^n \times_{M^n} M^k$ for various n, k with $n > k$ and the diagonal maps and the shriek maps between them.*

Though our main concern is the ordinary homology, the use of symmetric spectra is technically inevitable. Our construction is similar to that of the Cohen-Jones isomorphism in string topology, and we need to deal with higher homotopy concerning shriek maps. Symmetric spectra is suitable to this kind of problem.

References

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[2] P. Boavida de Brito, M. Weiss *Manifold calculus and homotopy sheaves*, preprint, arXiv:1202.1305 (2012).

[3] V. Turchin, *Context-free manifold calculus and the Fulton-MacPherson operad*, Algebr. Geom. Top., **13**, no.3, (2013) 1243-1271.

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