We discuss gluon scattering amplitudes of the four-dimensional maximally supersymmetric Yang-Mills (SYM) theory at strong coupling. By the gauge/string duality, the amplitudes are given by the area of the minimal surfaces in anti-de Sitter (AdS) space with a null-polygonal boundary. The area is in turn obtained through the Stokes data for the associated Hitchin system, which solve the thermodynamic Bethe ansatz equations describing finite-size effects of two-dimensional integrable models. Based on these interesting connections among four-dimensional SYM, ten-dimensional string theory on AdS and two-dimensional integrable systems, we derive analytic expansions of the amplitudes at strong coupling around certain kinematic configurations.