

A RELATION BETWEEN THE SPACES OF LONG KNOTS AND PURE BRAIDS

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The aim of this research is to understand the topology of the space $\mathcal{K}_n$ of *long knots* in $\mathbb{R}^n$, $n > 3$. In this talk we define a ‘capping map’ c from the space $\Omega\text{Conf}(\mathbb{R}^{n-1}, k)$ of pure braids in $\mathbb{R}^n$ to $\mathcal{K}_n$, and determine the induced maps on (co)homology groups. The following relies on the study on the loop space of configuration space [2, 3].

Theorem ([4]). The map c detects the (co)homology classes of $\mathcal{K}_n$ which are ‘higher dimensional analogues’ of finite type invariants for long knots in $\mathbb{R}^3$. $\square$

More precisely, $\text{Im}(c_*) \subset H_*(\mathcal{K}_n)$ is ‘dual’ to the subspace of de Rham cohomology classes obtained via the *configuration space integral* [1], a higher dimensional analogue of an integral expression of finite type invariants.

Recently P. Salvatore [5] proved that $\mathcal{K}_n$ ($n > 3$) is weakly equivalent to a two-fold loop space. Hence the map c can be extended to that from the double Freudenthal suspension. The extended map might produce some torsion elements of $H_*(\mathcal{K}_n)$.

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