The homotopy of spaces of algebraic maps between real algebraic varieties

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The main purpose of this talk is to talk about the recent joint work with M. Adamaszek and A. Kozlowski [2]. We consider the inclusion of the space $\mathrm{Alg}(X, Y)$ of algebraic (regular) maps between real algebraic varieties in the space $\mathrm{Map}(X, Y)$ of all continuous maps. For a certain class of real algebraic varieties X and Y, which include real projective spaces, it is known that $\mathrm{Alg}(X, Y)$ is a dense subspace in $\mathrm{Map}(X, Y)$. In this talk, as the first step, for certain class of varieties X and Y, we explain that the inclusion $\mathrm{Alg}(X, Y) \to \mathrm{Map}(X, Y)$ is also a homotopy equivalence. Next, we restrict the class of varieties to real projective spaces. In this case, the space of algebraic maps has a 'minimum degree' filtration by finite dimensional subspaces and it is natural to expect that the homotopy types of the terms of the filtration approximate closer and closer the homotopy type of the space of continuous mappings as the degree increases. This type of the conjecture is called as the Atiyah-Jones-Segal type conjecture or Gromov's h-principle (cf. [1], [3], [4], [7], [8], [9], [10], [11], [14], [15], [16], [17], [19]). We explain that this type result holds for this case and we compute the lower bounds of this approximation degree of these spaces.

References

[1] M. F. Atiyah and J. D. S. Jones, Topological aspects of Yang-Mills theory, Commun. Math. Phys. **61** (1978), 97–118.

[2] M. Adamaszek, A. Kozlowski and K. Yamaguchi, The space of algebraic and continuous maps between real algebraic varieties, preprint.

[3] C. P. Boyer, J. C. Hurtubise, B. M. Mann and R. J. Milgram, The topology of instanton moduli spaces I: The Atiyah-Jones conjecture, Ann. Math. **137** (1993), 561–609.

[4] C. P. Boyer, J. C. Hurtubise, B. M. Mann and R. J. Milgram, The topology of the space of rational maps into generalized flag manifolds, Acta Math. **173** (1994), 61–101.

[5] F. R. Cohen, R. L. Cohen, B. M. Mann and R. J. Milgram, The topology of rational functions and divisors of surfaces, Acta Math. **166** (1991), 163–221.

[6] R. L. Cohen, J. D. S. Jones, G. B. Segal, Stability for holomorphic spheres and Morse Theory, Contemporary Math. **258** (2000), 87–106.

[7] M. Gromov, Oka's principle for holomorphic sections of elliptic bundles, J. Amer. Math. Soc. **2** (1989) 851-897

[8] M. A. Guest, Topology of the space of rational curves on a toric variety, Acta Math. **174** (1995), 119–145.

[9] M. A. Guest, A. Kozlowski, and K. Yamaguchi, The topology of spaces of coprime polynomials, Math. Z. **217** (1994), 435–446.

[10] M. A. Guest, A. Kozlowski and K. Yamaguchi, Spaces of polynomials with roots of bounded multiplicity, Fund. Math. **116** (1999), 93–117.

[11] A. Kozlowski and K. Yamaguchi, Topology of complements of discriminants and resultants, J. Math. Soc. Japan **52** (2000), 949–959.

[12] A. Kozlowski and K. Yamaguchi, Spaces of holomorphic maps between complex projective spaces of degree one, Topology Appl. **132** (2003), 139–145.

[13] A. Kozlowski and K. Yamaguchi, Spaces of algebraic maps from real projective spaces into complex projective spaces, preprint.

[14] B. M. Mann, R. J. Milgram, Some spaces of holomorphic maps to complex Grassmann manifolds, J. Diff. Geom. 33 (1991), 301-324.

[15] J. Mostovoy, Spaces of rational loops on a real projective space, Trans. Amer. Math. Soc. **353** (2001), 1959–1970.

[16] J. Mostovoy, Spaces of rational maps and the Stone-Weierstrass Theorem, Topology **45** (2006), 281–293.

[17] G. B. Segal, The topology of spaces of rational functions, Acta Math. **143** (1979), 39–72.

[18] V. A. Vassiliev, Complements of Discriminants of Smooth Maps, Topology and Applications, Amer. Math. Soc., Translations of Math. Monographs **98**, 1992 (revised edition 1994).

[19] K. Yamaguchi, Complements of resultants and homotopy types, J. Math. Kyoto Univ. **39** (1999), 675–684.

[20] K. Yamaguchi, The homotopy of spaces of maps between real projective spaces, J. Math. Soc. Japan **58** (2006), 1163–1184; *ibid.* **59** (2007), 1235–1237.